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CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

59. Proposed by **MOSES COBB STEVENS, M. A.**, Department of Mathematics, Purdue University, Lafayette, Ind.

Solve $n\dfrac{d^2y}{dx^2}(x^2+y^2)^{\frac{1}{2}}=\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$.

[From *Forsyth's Differential Equations.*]

I. Solution by **WILLIAM HOOVER, A. M., Ph. D.**, Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

Let $y/x=z$, $d^2y/dx^2=y/x$; then the given equation becomes

$$nv\sqrt{1+z^2}=\{1+p^2\}^{\frac{3}{2}},$$

$$\text{or } v=\frac{\{1+p^2\}^{\frac{3}{2}}}{n\sqrt{1+z^2}},=(p-z)\frac{dp}{dz}\ldots\ldots\ldots\ldots\ldots\ldots(A).$$

Assuming $p=(z-t)/(1+zt)$, (A) reduces to

$$\frac{t(dt/dz)}{[1+t^2][t+(1/n)\sqrt{1+t^2}]}-\frac{1}{1+z^2}=0\ \ldots\ldots\ldots\ldots\ldots(B),$$

in which the variables are separated.

II. Solution by **G. B. M. ZERR, M. A., Ph D.**, President and Professor of Mathematics, Russell College, Lebanon, Va.

Let $x=r\cos\theta$, $y=r\sin\theta$, then the equation becomes,

$$nr^3+2nr\left(\frac{dr}{d\theta}\right)^2-nr^2\frac{d^2r}{d\theta^2}=\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{\frac{3}{2}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Let $\dfrac{dr}{d\theta}=p$, so that $\dfrac{d^2r}{d\theta^2}=p\dfrac{dp}{dr}$;

$\therefore$ (1) becomes, $nr^3+2nrp^2-nr^2p\dfrac{dp}{dr}=(r^2+p^2)^{\frac{3}{2}}$.

Let $p=r/v$, so that $\dfrac{dp}{dr}=\dfrac{1}{v}-\dfrac{r}{v^2}\dfrac{dv}{dr}$;

$\therefore$ (2) becomes, $nv(1+v^2)dr+nrdv=(1+v^2)^{\frac{3}{2}}\,dr$.

$$\therefore dr/r=\frac{ndv}{(1+v^2)\{(1+v^2)^{\frac{1}{2}}-nv\}}.$$

$$\therefore \log r=\log\left\{\frac{(1+v^2)^{\frac{1}{2}}}{(1+v^2)^{\frac{1}{2}}-nv}\right\}+\log A.$$

$$\therefore r=\frac{A(1+v^2)^{\frac{1}{2}}}{(1+v^2)^{\frac{1}{2}}-nv},=\frac{A(r^2+p^2)^{\frac{1}{2}}}{(r^2+p^2)^{\frac{1}{2}}-nr}.$$

$$\therefore r\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{\frac{1}{2}}-nr^2=A\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{\frac{1}{2}}$$

$$\therefore d\theta=\pm\frac{(r-A)dr}{r\sqrt{n^2r^2-(r-A)^2}},=\pm\frac{(r-A)dr}{nr^2\sqrt{1-\{(r-A)/(nr)\}^2}}.$$

$$\therefore \theta=\pm\frac{1}{\sqrt{n^2-1}}\log[(n^2-1)r+A+\sqrt{(n^2-1)\{r^2n^2-(r-A)^2\}}]$$

$$\mp\sin^{-1}\left(\frac{r-A}{nr}\right)+B.$$

$$\therefore \tan^{-1}(y/x)\pm\sin^{-1}\left(\frac{\sqrt{x^2+y^2}-A}{n\sqrt{x^2+y^2}}\right)-B,=\pm\frac{1}{\sqrt{n^2-1}}\log[(n^2-1)\sqrt{x^2+y^2}$$

$$+A+\sqrt{(n^2-1)\{n^2(x^2+y^2)-(\sqrt{x^2+y^2}-A)^2\}}],$$

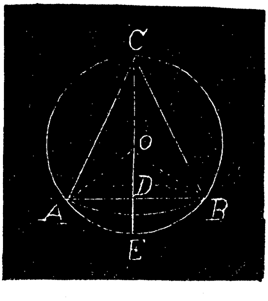
where A and B are constants of integration.

60. Proposed by SETH PRATT, C. E., Assyria, Mich.

To remove $(1/a)$th of the volume of a sphere of a given radius by a conical hole, whose axis is the axis of the sphere, and whose vertex is at the surface of the sphere. Required the height of the cone and the diameter of its base.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let O be the center of the sphere, ABC the cone, $AO=r$, $DE=x$, $AD=y$; then the volume of spherical segment



$$ABE=\frac{\pi x^2}{3}(3r-x),$$

and that of the cone

$$ABC=\frac{\pi y^2(2r-x)}{3};$$

by condition, therefore,